Modified Fourier Descriptors : A new parametrization of eddy current signatures applied to the rail defect classification

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Abstract

The work presented in this paper deals with the problem of parametrization of eddy current signals. In a defect recognition process, it is important to extract a reduced and relevant set of features that owns both a great descriptive potential of defects and a strong insensitivity to some transformations (offset, homothetic transformation and reverse description). An original parametrization procedure called Modified Fourier Descriptors is presented and compared to an autoregressive modeling. The last section considers the application of rail head inspection and some considerations about the parameter selection and the classification problem are detailed. The classification performances will be given for the two parametrization procedures and it will be shown the superiority of the Modified Fourier Descriptors for our application.

Keywords : parametrization, Fourier Descriptors, AutoRegressive process, eddy current, rail head inspection, selection, classification.

1. Introduction

The eddy current technology is widely adopted in many industrial applications for its robustness and simple use. An eddy current sensor is generally able to made a basic detection of defects or distance (all or nothing). But, if the process requires more complex analysis, the sensor has to be associated with advanced signal processing procedures in order to make up for its lack of resolution and its low measurement range. Commonly, a defect recognition process including a parametrization procedure has to be elaborated. In this paper, we only detail parametrization of defect signal obtained by 1-D scanning.

The purpose of the parametrization is to accurately describe the sensor output signals by means of a reduced and relevant set of parameters. So, the first property required for the parametrization is a great descriptive potential. The parametrization must also be invariant under some transformations of signal (cf. figure 1) such as offset (due to unbalanced sensor), homothetic transformation (which corresponds to variation of measurement distance) and reverse description (a defect must be characterized by the same representation vector for the two possible scanning directions).

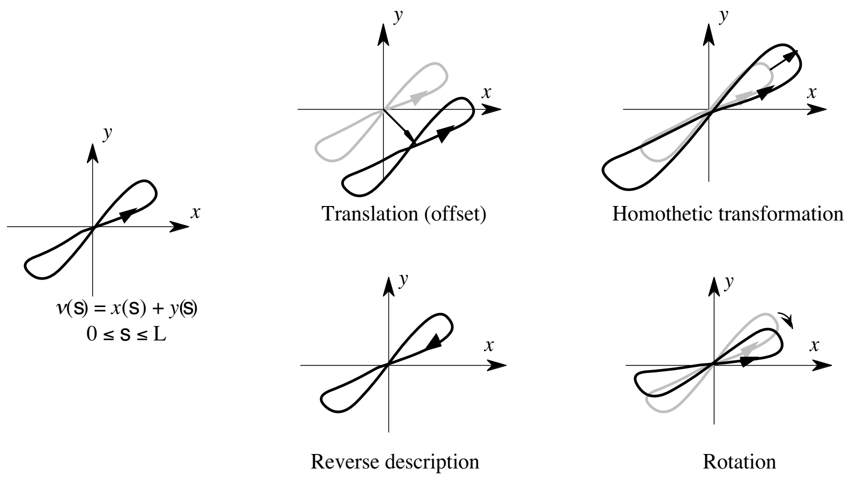

Figure 1 : Usual transformations on eddy current trajectories in impedance plane

The three transformations above mentionned must not affect the representation mode of defects. On the contrary, it is interesting that the parametrization of eddy current signatures depends on rotations in impedance plane that are linked to the electromagnetic losses in the target (cf. figure 1).

A lot of eddy current applications use heuristic methods as parametrization only based on expertise rules on signals [1]. This kind of approach consists of extracting a reduced number of parameters from the timing evolution of signals or from theirs spectrums. In time domain, the signal can be described by parameters such as peak to peak value, half-heigth width, signal energy, etc. The parameters deduced from the frequential domain may include maximum frequency response, the frequency of maximum energy, etc. The defect characterisation can also be based on properties of trajectories in impedance plane such as lobe area, orientation.... In practice, in order to get a relevant information about the defect, a combination of different sorts of parameters is often used to constitute the representation mode of signals.

The principal drawback of heuristic methods is that the result of defect recognition processus closely depends on the quality of the expertise. Particularly, if the expertise is not elaborated from all kinds of expected defect signatures or experimental conditions, the generalization capabilities are restricted. In order to avoid this drawback, other approachs consist of prior modeling of eddy current signals or trajectories.

2. Fourier Descriptors and their normalizations

2.1. Fourier Descriptors

The modeling of eddy current signals by the Fourier Descriptors uses the signature of defect in the impedance plane. The eddy current trajectory can be represented by a complex function defined by (cf. figure 1) :

$$v(s) = x(s) + j\, y(s) \qquad 0 \leq s \leq L \qquad (1)$$

where x and y are the in-phase and quadrature components of the measurement signal, L the total observation length of the defect.

In order to work on unlimited domain, a periodic signal $u(s)$ can be associated to the complex signal $v(s)$. The periodic nature of $u(s)$ allows us to expand it in a complex Fourier series :

$$u(s)=\sum_{k=-\infty}^{+\infty} f_k\, e^{j2\pi k\, s/L} \quad \text{with} \qquad f_k = \frac{1}{L}\int_{(L)} u(s)\, e^{-j2\pi k\, s/L}\, ds \qquad (2)$$

Fourier Descriptors f_k completely describe the eddy current trajectory and they constitute a possibility of parametrization. But they are sensitive to transformations on signal as translation, homothetic transformation and reverse description. Table 1 sums up the influence of these operations.

	Signal $u(s)$	Fourier Descriptors f_k
translation	$u(s)+u_0$	$f_0+u_0\ , f_k$
homothetic transformation	$\lambda\, u(s)$	$\lambda\, f_k$
reverse description	$u(L-s)$	$e^{-j\,2\,\pi\,k} f_{-k}$
rotation	$e^{j\varphi}\, u(s)$	$e^{j\varphi} f_k$

Table 1 : Influence of transformations on Fourier Descriptors

2.2. Granlund Descriptors

For character recognition application, Granlund proposed an alternative description by using non linear combinations of Fourier coefficients that become normalized [2] :

$$b_k = \frac{f_{1+k}\, f_{1-k}}{f_1^{\,2}} \qquad k = 2,3,\ldots \qquad \text{and} \qquad b_1 = \frac{f_2\,|f_1|}{f_1^{\,2}} \tag{3}$$

The Granlund descriptors b_k contain information about shape and are invariant under different operations such as translation, homothetic transformation. Only b_1 is sensitive to rotation of contour (this information is useful in character recognition to distinguish between characters that only differ in orientation as 6 and 9).

However, these descriptors are sensitive to reverse description on the signal. In fact, if the reverse signal is called *u'(s)* with

$$u'(s) = u(L-s)$$

the Fourier Descriptors of the reverse signal *u'(s)* are given by :

$$f'_k = e^{-j2\pi k}\ f_{-k}$$

Then, the Granlund Descriptors associated to *u'(s)* can be written as :

$$b'_k = \frac{f'_{1+k}\ f'_{1-k}}{f'^{\,2}_1} = \frac{e^{-j2\pi(1+k)}\ f_{-(1+k)}\ e^{-j2\pi(1-k)}\ f_{-(1-k)}}{e^{-j4\pi}\ f_{-1}^{\,2}} = \frac{f_{-(1+k)}\, f_{k-1}}{f_{-1}^{\,2}} \neq b_k$$

and

$$b'_1 = \frac{f'_2\,|f'_1|}{f'^{\,2}_1} = \frac{e^{-j4\pi}\ f_{-2}\ \left|e^{-j2\pi}\ f_{-1}\right|}{e^{-j4\pi}\ f_{-1}^{\,2}} = \frac{f_{-2}\,|f_{-1}|}{f_{-1}^{\,2}} \neq b_1$$

The Granlund descriptors are not easily usable in eddy current NDT applications because they are modified with reverse description. So, we have elaborated a new parametrization called Modified Fourier Descriptors.

2.3. Modified Fourier Descriptors (MFD)

From the Fourier Descriptors, the Modified Fourier Descriptors (MFD) are defined by :

$$d_k = \frac{f_k\ f_{-k}}{\left|f_1\, f_{-1}\right|} \qquad k = 1,2,3,\ldots \tag{4}$$

For the particular point of reverse contour, the MFD are given by :

$$d'_k = \frac{f'_k\ f'_{-k}}{\left|f'_1 f'_{-1}\right|} = \frac{e^{-j2\pi k}\ f_{-k}\ \ e^{j2\pi k}\ f_k}{\left|e^{-j2\pi} f_{-1}\ \ e^{j2\pi}\ f_1\right|} = \frac{f_k\ f_{-k}}{\left|f_1\ f_{-1}\right|} = d_k \tag{5}$$

As we show in equation (5), the Modified Fourier Descriptors are invariant under reverse description of contour. It can also be shown that they are invariant under offset and homothetic transformation [3] and they remain sensitive to rotation of contour.

	Signal $u(s)$	Granlund Descriptors b_k	Modified Fourier Descriptors d_k
translation	$u(s)+u_0$	b_k	d_k
homothetic transformation	$\lambda\ u(s)$	b_k	d_k
reverse description	$u(L-s)$	$b'_k \neq b_k$	d_k
rotation	$e^{j\varphi}\ u(s)$	$b_1 e^{j\varphi}$, b_k	$e^{j2\varphi}\ d_k$

Table 2 : Influence of transformations on Granlund Descriptors and Modified Fourier Descriptors

Table 2 compares the influence of the transformations on Granlund Descriptors and Modified Fourier

Descriptors. The parametrization procedure must be choosen in accordance with the problem invariants that may not affect the representation mode of defects.

3. AutoRegressive model

In the preceding paragraph, we looked at some approaches for modeling the eddy current trajectories in impedance plane. In this section, we consider the usual modeling of the time evolution of eddy current signals in terms of autoregressive process. So, the in-phase and quadrature components are separately processed.

A real measurement signal *x(n)* is modeled as a *p* order AutoRegressive (AR) process, generated by filtering a unit variance white noise *b(n)* :

$$x(n) = -\sum_{k=1}^{p} a_k \, x(n-k) + b(n) = \hat{x}(n) + b(n) \qquad (6)$$

Therefore, the signal is described by the model parameters a_k. The coefficient vector $\underline{a}$ are given by the normal equation [4] :

$$\underline{a} = -R^{-1}\underline{r} \qquad \text{where} \qquad (7)$$

$\underline{r} = \left[r_1, \; r_2, \; \cdots \; r_p \right]^t$ and $R = \begin{bmatrix} r_0 & r_1 & \cdots & r_{p-1} \\ r_1 & r_0 & \cdots & r_{p-2} \\ \vdots & & & \vdots \\ r_{p-1} & \cdots & & r_0 \end{bmatrix}$ are the correlation function and matrix of x.

Because of the stationnarity of the process during each defect scanning, the elements of $\underline{r}$ and R are given by

$$r_i = \mathrm{E}\left[x(n) \; x(n-i) \right]$$

For the AR coefficient estimation, the direct solution of (7) leads to high computational cost. The minimizing of the cost function J is usually preferred where J computes the mean square error between the signal and its approximation $\hat{x}(n)$:

$$J = \frac{1}{N} \sum_{n=1}^{N} \left(x(n) - \hat{x}(n) \right)^2$$

with N the data record length.

For the minimization of J, several methods are available such as autocorrelation method or covariance method [4].

To select the appropriate model order p of the AR process, the use of cost function J is not suitable because J is monotonically decreasing with the model order; any minimum value of J can be found. Then, other criterions have been proposed [4] that include a penalty function which increases with the model order, as the Akaïke information criterion,

$$AIC(J, \underline{a}) = N \; \ln(J(\underline{a})) + 2\,p \qquad (8)$$

The advantage is that the evolution of the criterion (8), function of the model order, presents a minimum value which corresponds to an appropriate model order.

The influence of the transformations described in figure 1, remains to be studied. In this paper, we detail the problem of reverse description. Let *x'(n)* be the reverse description of the initial signal *x(n)* :

$$x'(n) = x(N-n) \qquad \forall \;\; 1 \le n \le N$$

The AR coefficients associated to $x'(n)$ may be written as :

$$\underline{a}' = -R'^{-1}\,\underline{r}'$$

The elements of R' and r' are $\quad r_i' = \mathrm{E}\big[x'(n)\; x'(n-i)\big] = \mathrm{E}\big[x(N-n)\; x(N-n+i)\big]$

The stationarity of the process implies that the correlation only depends on the absolute difference between the indices. So

$$r_i' = r_{-i} = r_i$$

Thus, the AR coefficients are similar for the signal $x(n)$ and its reverse description $x'(n)$.

$$\underline{a'} \;=\; -R^{-1}\,\underline{r} \;=\; \underline{a}$$

Similarly, we can show that neither offset nor homothetic transformation affect the autoregressive parametrization [3].

However, as this AR modeling dissociates the in-phase and quadrature components, the parametrization becomes independant from the rotation in the complex plane that is still useful to discriminate the different classes of defects in eddy current applications.

4. Application to the rail defect classification

4.1. Experimental tests

These modeling developments have been tested on eddy current signals obtained from a dedicated sensor mounted underneath a commercial railway car, for in-situ rail inspection. The measurement distance was 20mm and the inspection velocity raised up to 70km/h. The devive uses both a differential and a double-differential coils powered by a bi-frequency current supply. So, the outputs of the sensor made up of 4 complex signals (8 real signals) that have to be modeled.

As indicated in the specification sheet, the defect classes are :

Class 1 : transversal cracks,
Class 2 : welded joints,
Class 3 : spalls of the rail head or shellings,
Class 4 : fishplated joints before switch

The last class of defect must be taken into account because of the switch crossing induces a particular magnetic to amagnetic target transition.

A data base of 140 defects has been constituted to elaborate the complete defect recognition system.

4.2. Parameter Selection

The parametrization of all eddy current signals is performed by one of the methods above described : Modified Fourier Descriptors or Autoregressive coefficients. The parametrization of the 4 output signals leads to a set of about 100 parameters. This dimension is too large for effective classification.

Among all the available parameters, it is important to keep the only relevant parameters related to the class separability. The redundant and/or irrelevant parameters will be simply ignored, thereby leading to a less complex classifier achievement. This parameter selection was carried out by means of a Gram-Schmidt orthogonalization procedure [5] that arranges the parameters from the most significant to the least one in terms of contribution to the class separability. Only the ten or twenty first parameters will be kept.

One possible measuring method of the class separability is to analyse the "geometry" of the class in the representation space. Two points have to be combinated : the compacity of the classes around their gravity center and the distance between the gravity centers (cf. figure 2).

The ratio of the within-class scatter matrix V_w to the between-class scatter matrix V_b is a suitable criterion

[6] :

$$\xi \;=\; \mathrm{trace}\left(V_w^{-1}\;V_b\right) \tag{9}$$

The more ξ is large, the more the classes are well-separated.

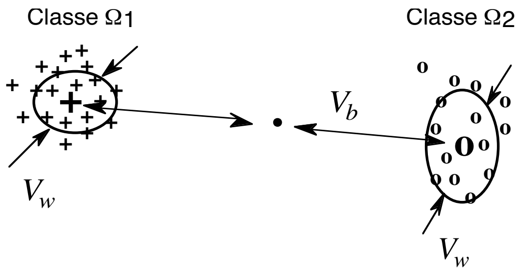

Figure 2 : Geometry of the classes in 2D representation space

A partionning approach of the classification problem is preferred rather than a global approach [7] in which the multiclass problem is solved simultaneously. Then, the K class initial problem (in our case K=4) is splitted into K sub-problems; each sub-problem is dedicated to the separation of one class among the others $\{\text{Class i / all classes}\}_{1\leq i\leq 4}$. This approach allows an independance of the design of the sub-classifiers.

For the 4 classification sub-problems, figure 3 compares the evolution of the criterion (9) for the two parametrization methods when the representation space dimension varies.

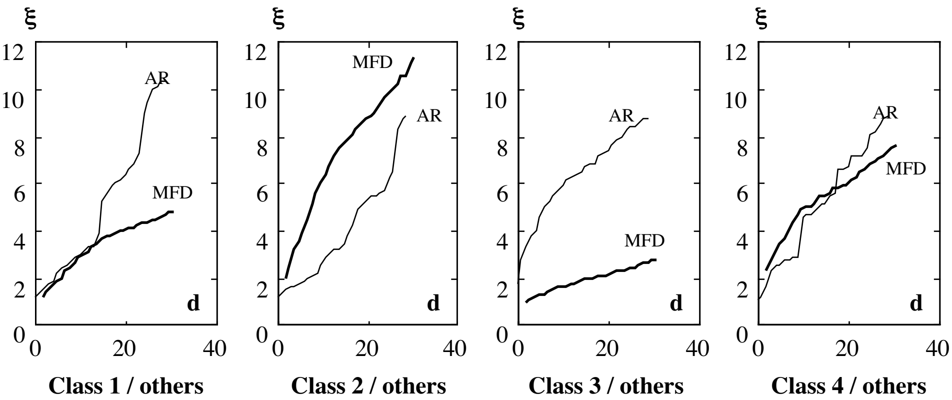

Figure 3 : Evolution of the criterion ξ function of the dimension of the representation space for the 4 sub-problems of classification

For small value of d, the MFD seem to be more efficient than the AR Descriptors except for the third sub-problem. This point will be highlighted on the performances of classification procedure.

4.3. Results of defect classification

We have choosen to perform the classification procedure by means of Neural Networks using the partionning approach. This attractive pre-processing of the defect signatures leads to simply linear classifiers (a single neuron for each elementary classifier). The activation function of each neuron is a sigmoïde varying between 0 to 1 that straight delivers the posterior class probability. Then, the defect is assigned to the maximum probability class. The input weigths were adjusted during a learning phase that computes the gradient descent, estimated from the above-mentionned data base [8]. Table 3 summarizes the 4 sub-classifiers and global performance obtained from the AR or MFD descriptors with the "leave one out" method [8].

Without reject class, the MFD give better global results than the AR Descriptors. Only for the particular third sub-problem, the performances confirm the superiority of AR approach (cf. §4.2).

These results can be improved by a "2 by 2" partitioning of the problem (each class is opposed to another). Then, the global performances reach 95% [3].

	AR	MFD
Classe 1 / others	87.8%	91.4%
Classe 2 / others	85.7%	95.7%
Classe 3 / others	98.5%	95.7%
Classe 4 / others	96.4%	97.8%
Global performance	80%	93.5%

Table 3 : Performances of the linear classifiers

5. Conclusion

Two prametrization methods were detailed in this paper : an original parametrization procedure called Modified Fourier Descriptors and the classical signal modeling of autoregressive type. For these two types of parametrization, both the principle and the influence of transformations listed as invariants of the problem (offset, homothetic transformation and reverse description) were exposed. However, except for the transformation influence, the relevance of one set of descriptors can not be carried out without the evaluation of the complete classification procedure.

For the application of inspection of the rail head, a defect recognition process including a parameter selection and a neuronal classification is described in the last section. The classification performances obtained for the two approaches show that the Modified Fourier Descriptors bear more relevance to our application. This new approach stands for an attractive tool for the parametrization of eddy current signatures, particularly if an insensitivity to reverse description of signals is required.

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